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Acoustic enhancement of heat transfer between two parallel plates

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Abstract—The paper considers a problem in which a steady-state sonic wave is propagated in the longitudinal direction in a fluid enclosed between two horizontal parallel plates which are kept at different temperatures. The distance between the plates is much smaller than the sound wavelength. Rayleigh's vortical acoustic streaming that appears in the region between the plates as a result of the sound wave leads to forced heat convection. The effect of that forced convection on heat transferred between the plates is analyzed theoretically. An acoustic Peclet number, which represents the interaction between heat conduction and forced convection is introduced, and asymptotic relations expressing the mean Nusselt number in terms of this dimensionless group are derived. The results obtained demonstrate that acoustic streaming results in a marked enhancement of heat transfer between the plates.

INTRODUCTION

The study of acoustic streaming was started with the work of Lord Rayleigh [1], who considered the vortex flow which occurs in a long pipe as a result of the presence of a longitudinal standing wave. This work was continued by Westervelt [2], Nyborg [3] and Schlichting [4]. Lighthill [5] has emphasized the fundamental role of dissipation of the acoustic energy in the evolution of the gradients in the momentum flux, which bring about the secondary streaming. Stuart [6] has introduced the streaming Reynolds number, R_S, based on the characteristic velocity of the secondary flow. In contrast to the previous investigations which considered $R_S \ll 1$, he studied the cases where $R_S \gg 1$.

The heat transfer process in the pulsating pipe flow was examined by Romie [7]. Rott [8] has investigated the acoustic oscillations in an infinite fluid region near a flat plate. The effect of mean temperature variation along the direction of oscillations is included. Wang and Kassoy [9] have studied the thermoacoustic process in a shear flow contained between two rigid parallel plates. The results describe the general transient evolution of acoustic waves driven by a plane source located at a given duct cross-section.

The effect of sound waves on both natural and forced convection from a cylinder has been studied extensively. References [10–13] provide an overview of the works done in this field. The works of Parker and Welsh [14] and Cooper *et al.* [15] deal with experiments that consider the influence of sound waves on forced convection from horizontal flat plates. The effect of sound on natural convection for a vertical flat plate has been studied by Engelbrecht and Pretorius [16].

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The effect of an oscillating flow field on heat and mass transfer from single spherical particles and droplets has also been investigated. Some examples of these theoretical studies can be found in refs. [17–22]. These publications report increases, decreases or unnoticeable changes in heat and mass transfer, depending on the frequency and magnitude of the steady and oscillating flow.

The present study considers the effect of Rayleigh's vortical acoustic streaming on heat transfer between two horizontal parallel plates which are kept at different constant temperatures. The condition $R_S \ll 1$ for the streaming Reynolds number is assumed to be satisfied throughout the domain.

FORMULATION OF THE PROBLEM

One of the most interesting ways in which sound waves are affected by viscosity is in the formation of steady vortex flow around solid obstacles or near boundaries. This acoustic streaming occurs in the second approximation with respect to the wave amplitude; its characteristic feature is that the velocity in it, namely, in the region outside a thin periodic boundary layer is independent of the viscosity, even though it originates from that viscosity [1].

The properties of acoustic streaming are most typically seen when the characteristic length in the problem (in the present case the distance between the parallel plates) is much smaller than the sound wavelength λ, but much larger than the thickness of the periodic boundary layer (the penetration depth), $l = \sqrt{2\nu/\omega}$ for viscous waves:

$$\lambda \gg h \gg l.$$

In view of the latter condition, we can distinguish

NOMENCLATURE

- a thermal diffusivity
- c_0 speed of sound in a fluid
- F_1, F_2 the integrals defined in equations (28) and (32)
- h distance between the plates
- k thermal conductivity
- l thickness of periodic boundary layer
- $\overline{Nu}$ mean Nusselt number
- n wave number
- p pressure
- $Pe = t_a/t_c$ acoustic Peclet number
- q heat flux
- R_s streaming Reynolds number
- T temperature
- T_1, T_2 the temperatures of the plates
- t time
- t_a characteristic time connected to thermal diffusivity
- t_c characteristic time connected to forced convection
- U velocity outside periodic boundary layer
- $v_c = h/t_c$ characteristic velocity of forced convection
- v_x, v_y velocity components
- $v_x^{(1)}, v_x^{(2)}$ successive approximations of the velocity
- v_0 amplitude of sonic wave.

Greek symbols

- δ characteristic dimension of the temperature change region
- $\theta^{(0)}, \theta^{(1)}, \theta^{(2)}$ successive approximations of the temperature
- λ wavelength
- ν dynamic viscosity
- ψ stream function
- ω frequency.

Superscript

- $'$ dimensional variable.

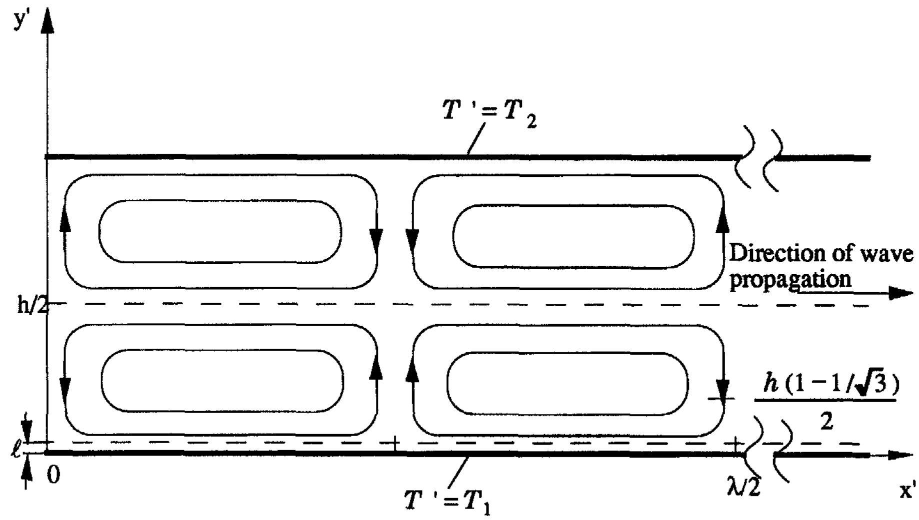

Fig. 1. Rayleigh's acoustic streaming between two horizontal plates.

in the flow region a narrow acoustic boundary layer in which the velocity decreases from its value in the sound wave to zero at the solid surface. Since the velocity in this layer, as in the sound wave itself, is much less than that of sound, and the characteristic dimension l is much less than λ, the flow there may be regarded as incompressible.

We now consider the problem of acoustic streaming between the horizontal walls, $y' = 0$ and $y' = h$ (Fig. 1) which are kept at different constant temperatures, T_1 and T_2, $T_2 > T_1$. The present analysis assumes that the effects of natural convection are negligible.

Let us consider the acoustic boundary layer at the x'–z' plane on one of the solid walls, assuming two-dimensional flow in the x'–y' plane (Schlichting [4]):

$$\frac{\partial v_x'}{\partial t'} + v_x' \frac{\partial v_x'}{\partial x'} + v_y' \frac{\partial v_x'}{\partial y'} - \nu \frac{\partial^2 v_x'}{\partial y'^2} = U' \frac{\partial U'}{\partial x'} + \frac{\partial U'}{\partial t'}. \quad (1)$$

In the present case the flow velocity $U'(x', t')$ outside the boundary layer is given by

$$U' = v_0 \cos nx' \cos \omega t' = v_0 \cos nx' \,\mathrm{re}\, e^{-i\omega t'} \quad (2)$$

where $n = \omega/c_0$, which corresponds to a plane stationary sound wave with frequency ω. Equation (1) can be solved by successive approximations with respect to the small parameter v_0, i.e. the amplitude of the velocity fluctuations in the sound wave.

The first approximation $v_x'^{(1)}$ satisfies the linear differential equation

$$\frac{\partial v_x'^{(1)}}{\partial t} - \nu \frac{\partial^2 v_x'^{(1)}}{\partial y^2} = -i\omega v_0 \cos nx' \, e^{-i\omega t'}. \quad (3)$$

The calculations performed by Schlichting [4] yield

$$v_x'^{(1)} = \mathrm{re}[v_0 \cos nx' e^{-i\omega t'}(1 - e^{-\sqrt{-(i\omega/\nu)}y'})]. \quad (4)$$

The second approximation is obtained as

$$\frac{\partial v_x'^{(2)}}{\partial t} - \nu \frac{\partial^2 v_x'^{(2)}}{\partial y^2} = U' \frac{\partial U'}{\partial x} - v_x'^{(1)} \frac{\partial v_x'^{(1)}}{\partial x} - v_y'^{(1)} \frac{\partial v_x'^{(1)}}{\partial y}. \quad (5)$$

The right-hand side of equation (5) contains terms with frequencies $\omega + \omega = 2\omega$ and $\omega - \omega = 0$. The latter give rise to time-independent terms, which are the ones representing the steady flow in question; we consider only this part of the velocity. Thus there is a steady-state component in the solution which does not vanish at large distances from the wall, i.e. outside the acoustic boundary layer. Its magnitude is

$$v_x'(\infty) = \frac{3v_0^2}{8c_0} \sin 2nx'. \quad (6)$$

One can see that outside the boundary layer there is, in the second approximation with respect to v_0, a steady flow, the velocity of which is independent of the viscosity. The velocity at the edge of the boundary layer, as given by equation (6) serves as a boundary condition for determining the main acoustic flow. In the case of a small streaming Reynolds number, R_S, introduced by Stuart [6], in which the velocity is the steady state velocity of the secondary flow, as given in equation (6), the acoustic streaming can be described by the Navier–Stokes equations over the whole region considered, $0 < y' < h$. Since the velocity of the required steady flow is much lower than that of sound, the flow may be regarded as incompressible. Moreover, since v_0 is assumed infinitesimal in the sound wave, the quadratic terms in the equation of motion may be neglected. Hence, the Navier–Stokes equations, rewritten in terms of the stream function, are reduced to (Landau and Lifshits [23])

$$\Delta^2 \psi' = \left(\frac{\partial^2}{\partial x'^2} + \frac{\partial^2}{\partial y'^2}\right)^2 \psi' = 0. \quad (7)$$

Because of the condition $h \ll \lambda$, the derivatives with respect to y' are much larger than those with respect to x'. Neglecting the latter, and using the boundary conditions of equation (6) at $y' = 0$, $y' = h$, and the obvious symmetry of the problem about the plane $y' = h/2$ one obtains the solution [1]

$$v_x' = -\frac{3v_0^2 \sin 2nx'}{16c_0}\left[1 - \frac{3(y'-h/2)^2}{(h/2)^2}\right]$$

$$v_y' = \frac{3nv_0^2 \cos 2nx'}{8c_0}\left[\left(y' - \frac{h}{2}\right) - \frac{(y'-h/2)^3}{(h/2)^2}\right]. \quad (8)$$

Rayleigh's acoustic streaming described by these expressions consists of two series of vortices located symmetrically about the median plane $y' = h/2$ and periodic in the x-direction, with a period $\lambda/2$ as shown in Fig. 1. The direction of the flow inside the vortices is indicated in the figure by arrows. The x-component of the velocity, v_x', changes sign at a distance $h(1-1/\sqrt{3})/2$ from the wall.

The energy equation with the relevant boundary conditions that describes the heat transfer in the region considered is

$$v_x' \frac{\partial T'}{\partial x'} + v_y' \frac{\partial T'}{\partial y'} = a \frac{\partial^2 T}{\partial y'^2}$$

$$T' = T_1 \quad @ \quad y' = 0 \quad T' = T_2 \quad @ \quad y' = h$$

$$\frac{\partial T'}{\partial x'} = 0 \quad @ \quad x' = 0 \quad x' = \frac{\lambda}{2} \quad (9)$$

where v_x', v_y' are given by equation (8).

The boundary conditions at $x' = 0$, $x' = \lambda/2$ imply that there is no external temperature gradient in the x-direction. One should bear in mind that we are dealing with time-averaged heat process since the velocity determining the convection is of the mean secondary flow.

ASYMPTOTIC TREATMENT AND RESULTS

We now define the various times connected with the process at hand

$$t_c = \frac{8c_0^2}{3v_0^2 \omega} \quad t_a = \frac{h^2}{4a} \quad (10)$$

where t_c is the characteristic time of the forced convection process and t_a the characteristic time of thermal diffusivity. Let the following dimensionless group be introduced:

$$Pe = \frac{t_a}{t_c} = \frac{3v_0^2 h^2 \omega}{32ac_0^2}. \quad (11)$$

This is essentially an acoustic Peclet number. When $Pe \ll 1$, i.e. the characteristic time of the convective process is relatively large, the influence of the acoustic streaming upon heat transfer is very small. On the other hand, when $Pe \gg 1$, the effect of the forced convection is dominant.

We now reformulate the problem in terms of the following dimensionless variables

$$x = 2nx', \quad y = \frac{2y'}{h} - 1, \quad \theta = \frac{T' - T_1}{T_2 - T_1}. \quad (12)$$

Equation (9) combined with equation (8) and together with the corresponding boundary conditions is rewritten as follows:

$$-(1-3y^2)\sin x\frac{\partial\theta}{\partial x}+(y-y^3)\cos x\frac{\partial\theta}{\partial y}=Pe^{-1}\frac{\partial^2\theta}{\partial y^2} \tag{13}$$

$$\theta=0 \quad @ \quad y=-1$$
$$\theta=1 \quad @ \quad y=1$$
$$\frac{\partial\theta}{\partial x}=0 \quad @ \quad x=0 \quad x=2\pi. \tag{14}$$

The acoustic streaming in terms of the dimensionless variables together with the boundary conditions of the thermal problem are shown in Fig. 2. Two different limiting cases for *Pe* values will be considered.

Case of Pe $\ll$ 1

Let us first consider the case when the influence of forced convection is very small, i.e. when $Pe \ll 1$. In this case the solution of equation (13) can be built by the method of the successive approximations. We seek the solution in the form

$$\theta=\theta^{(0)}+Pe\theta^{(1)}+Pe^2\theta^{(2)}+\ldots \tag{15}$$

Substitution of the expansions of equation (15) into equation (13) yields the zeroth approximation

$$\theta^{(0)}=\tfrac{1}{2}(y+1). \tag{16}$$

This solution describes the temperature distribution in the region without any influence of forced convection. For the first approximation equation (13) yields

$$\frac{\partial^2\theta^{(1)}}{\partial y^2}=\tfrac{1}{2}\cos x(y-y^3)\,;\theta^{(1)}=0 \quad @ \quad y=\pm 1. \tag{17}$$

The solution of equation (17) is

$$\theta^{(1)}=\tfrac{1}{2}\cos x\left(\frac{y^3}{6}-\frac{y^5}{20}-\frac{7y}{60}\right). \tag{18}$$

For the second approximation equations (13) and (14) yield

$$\frac{\partial^2\theta^{(2)}}{\partial y^2}=\tfrac{1}{4}(1-\cos 2x)(1-3y^2)\left(\frac{y^3}{6}-\frac{y^5}{20}-\frac{7y}{60}\right)$$
$$+\tfrac{1}{4}(1+\cos 2x)(y-y^3)\left(\frac{y^2}{2}-\frac{y^4}{4}-\frac{7}{60}\right)$$
$$\theta^{(2)}=0 \quad @ \quad y=\pm 1. \tag{19}$$

One notes that the right-hand side of equation (19) contains terms that do not depend on x. The latter give rise to x-independent terms in the solution, which are the ones representing a wall heat flux which is averaged with respect to x; we consider only this part of the solution. Omitting simple intermediate calculations we obtain for the mean temperature gradient at the wall

$$\overline{\left(\frac{\partial\theta}{\partial y}\right)}=\tfrac{1}{2}(1+0.007Pe^2) \quad @ \quad y=\pm 1 \tag{20}$$

and the mean Nusselt number

$$\overline{Nu}=1+0.007Pe^2\,,\quad Pe\ll 1. \tag{21}$$

As seen from equation (21) the effect of acoustically enhanced heat convection for this case is rather small. Moreover, the result obtained has only theoretical significance, since, when $Pe \ll 1$ the effect of natural convection, neglected in the present analysis, may be important in the presence of gravity.

Case of Pe $\gg$ 1

Let us now consider the more interesting case, when the effect of the acoustic field is very large, i.e. when $Pe \gg 1$. In this case, the coefficient of the highest order derivative in equation (13) is very small: therefore, heat conduction manifests itself only in the narrow regions in which both components of the velocity

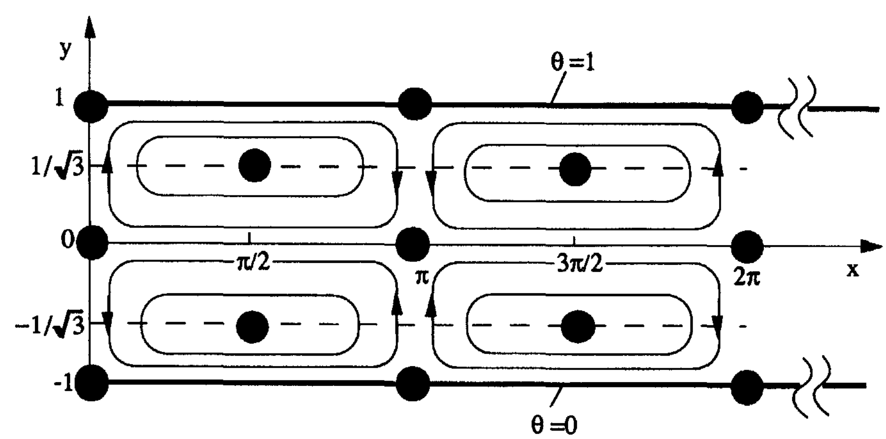

Fig. 2. Acoustic streaming between the plates in the dimensional variables. The small discs represent the areas of heat conduction effect.

vanish. These regions are noted in Fig. 2 by small black disks. It is clear *a priori*, that the contribution to the mean heat flow at the walls is due to heat conduction only in the vicinity of the points $(0, -1)$, $(0, +1)$, $(\pi, -1)$, $(\pi, 1)$, $(2\pi, -1)$ and $(2\pi, 1)$. As the problem is periodic with a period of 2π, the contributions at $x = 2\pi$ are identical to those at $x = 0$.

First of all we estimate the distance over which the temperature near the walls changes noticeably. For large Pe, the distance δ' is small compared with the distance between the plates, h. The order of magnitude of δ' may be obtained from an estimate of the terms in equation (9). Over the distance from $y' = 0$ (or $y' = h$) to $y' \sim \delta'$ (or $y' \sim h-\delta'$) at $x' = 0$ (or $x' = \lambda/4$), the temperature varies by an amount of the order of the total temperature difference $T_2 - T_1$ between the walls, while the velocity v_y' varies over the distance by an amount of the order of

$$v_c \frac{\delta'}{h} \sim \frac{\delta'}{t_c}, \quad v_c = \frac{h}{t_c} \tag{22}$$

where v_c is the characteristic value of convective velocity, v_y', since the total change of the order of v_c occurs over the distance h. Hence, for $y' \sim \delta'$ and $x' = 0$, the terms in equation (9) are, in order of magnitude,

$$a\frac{\partial^2 T'}{\partial y'^2} \sim a\frac{T_2 - T_1}{\delta'^2} \quad \text{and} \quad v_y'\frac{\partial T'}{\partial y'} \sim \frac{T_2 - T_1}{t_c}. \tag{23}$$

If the two expressions are comparable, we have

$$\delta' \sim \sqrt{at_c} \sim \sqrt{at_a Pe^{-1}} \sim hPe^{-1/2}. \tag{24}$$

Thus, for large Pe, the thickness of the temperature boundary region decreases inversely as the square root of Pe. The heat flux is given by

$$q = -k\frac{\partial T'}{\partial y'} \sim k\frac{T_2 - T_1}{\delta'} \sim \frac{k(T_2 - T_1)}{h} Pe^{1/2} \tag{25}$$

and the required limiting law of heat transfer is found to be

$$\overline{Nu} = \text{const}\, Pe^{1/2}. \tag{26}$$

In order to determine the value of the constant in equation (26), let us treat equations (13) and (14) in the vicinity of the lines $x = 0$ and $x = \pi$.

At $x = 0$ equations (13) and (14) yield

$$\frac{\partial^2 \theta}{\partial y^2} = Pe(y - y^3)\frac{\partial \theta}{\partial y}$$

$$\theta = 0 \quad @ \quad y = -1 \quad \theta = 1 \quad @ \quad y = 1. \tag{27}$$

The solution of the problem is given by

$$\theta = \frac{1}{F_1}\int_1^y e^{Pe(y^2/2 - y^4/4)}\, dy$$

$$F_1 = \int_{-1}^{1} e^{Pe(y^2/2 - y^4/4)}\, dy. \tag{28}$$

The integral F_1 at $Pe \gg 1$ may be estimated by the Laplace method [24]:

$$F_1 = \sqrt{\frac{\pi}{Pe}} \exp\left(\frac{Pe}{4}\right), \quad Pe \to \infty. \tag{29}$$

In accordance with equations (28) and (29), the temperature gradient near the walls $y = \pm 1$ is given by

$$\left.\frac{\partial \theta}{\partial y}\right|_{y=\pm 1} = \frac{1}{\sqrt{\pi}} Pe^{1/2}. \tag{30}$$

At $x = \pi$ equations (13) and (14) yield

$$\frac{\partial^2 \theta}{\partial y^2} = -Pe(y - y^3)\frac{\partial \theta}{\partial y}$$

$$\theta = 0 \quad @ \quad y = -1 \quad \theta = 1 \quad @ \quad y = 1. \tag{31}$$

The solution of the problem is given by

$$\theta = \frac{1}{F_2}\int_{-1}^y e^{-Pe(y^2/2 - y^4/4)}\, dy$$

$$F_2 = \int_{-1}^{1} e^{-Pe(y^2/2 - y^4/4)}\, dy. \tag{32}$$

The integral F_2 at $Pe \gg 1$ may also be estimated by the Laplace method:

$$F_2 = \sqrt{\frac{2\pi}{Pe}}, \quad Pe \to \infty. \tag{33}$$

The temperature gradient near the walls $y = \pm 1$ is given by

$$\left.\frac{\partial \theta}{\partial y}\right|_{y=\pm 1} = \sqrt{\frac{1}{2\pi}} Pe^{1/2} \exp\left(-\frac{Pe}{4}\right). \tag{34}$$

One could see that this term vanishes as Pe tends to infinity. Hence, at large Peclet numbers the mean heat flux is determined by heat transfer in the vicinity of the line $x = 0$, where the forced convective motion caused by the acoustic field is directed to the walls. From equation (30) it follows that the expression for mean Nusselt number has the form

$$\overline{Nu} = \frac{2}{\sqrt{\pi}} Pe^{1/2} = 1.13 Pe^{1/2}, \quad Pe \gg 1. \tag{35}$$

Thus, for large Peclet numbers the mean Nusselt number is proportional to the square root of Pe and the coefficient of proportionality is equal to $2/\sqrt{\pi}$. This means that $\overline{Nu}$ is proportional to the amplitude and to the square root of the frequency of acoustic oscillations.

Let us consider, for example, heat transfer in air, $a = 0.2 \times 10^{-4}$ m^2 s^{-1}, $c_0 = 330$ m s^{-1} between two parallel plates with $h = 0.1$ m, which is affected by stationary sound waves of various intensities and frequencies. The results are presented in Fig. 3. The considered intensities are of 140 dB and 145 dB. This example demonstrates the significant acoustic enhancement of heat transfer in a certain range of

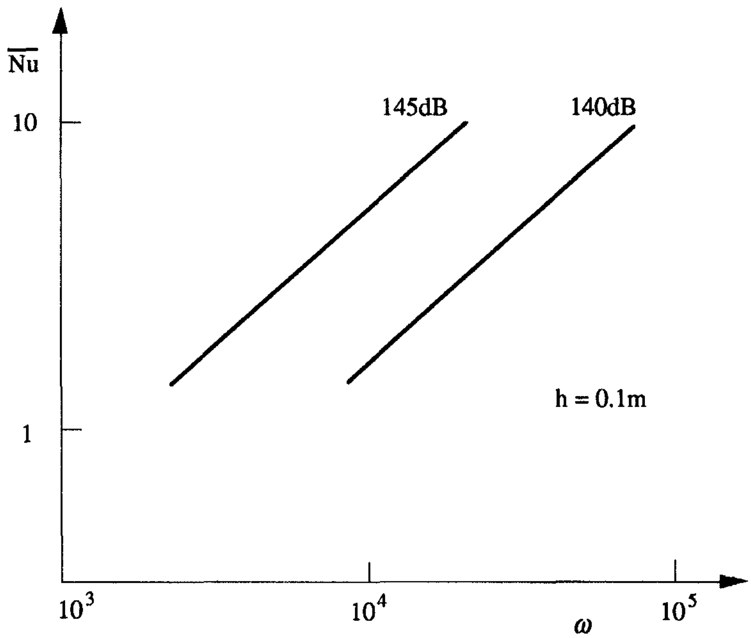

Fig. 3. Average Nusselt number as a function of frequency for two different sound wave intensities in air.

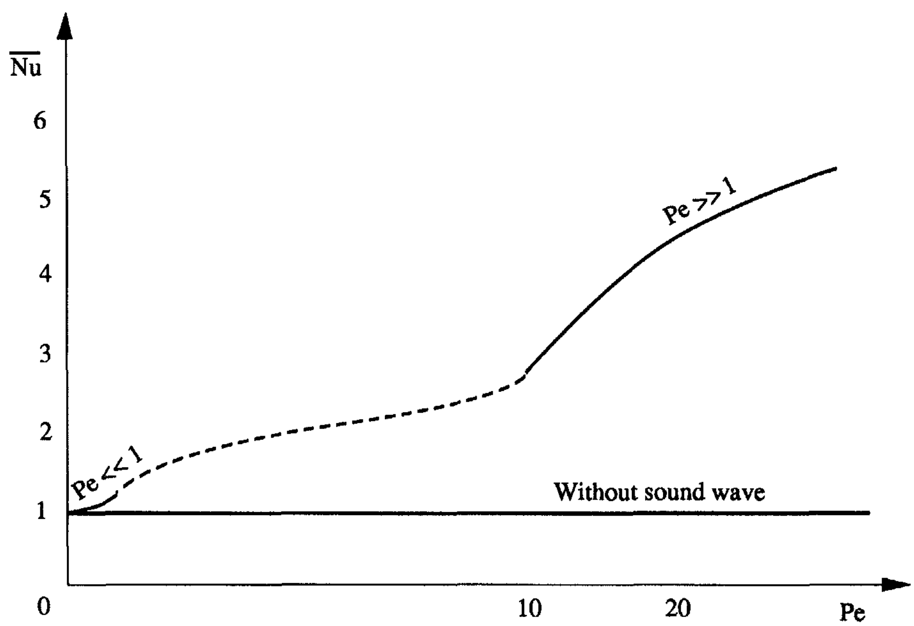

Fig. 4. Average Nusselt number as a function of *Pe*.

sound wave frequencies, especially in the ultrasonic range.

SUMMARY

A sonic wave which propagates longitudinally in a fluid between two parallel horizontal plates has been shown to enhance the heat transfer from the plates to the fluid. This heat transfer is a result of Rayleigh's vortical acoustic streaming. The steady state two-dimensional energy equation describing that forced convection is solved by the method of asymptotic expansions. The solution shows the effect of acoustic streaming on the heat transfer between the plates.

The problem of heat transfer in the presence of a sonic field is formulated in terms of the acoustic *Pe* number, which represents the interaction between heat conduction and forced convection. Asymptotic relations for $Pe \ll 1$ and $Pe \gg 1$ expressing the time and longitudinal direction averaged Nusselt number, $\overline{Nu}$, are derived. The qualitative dependence of $\overline{Nu}(Pe)$ is reproduced in Fig. 4. The present analysis applies only to small and large values of *Pe*. The solution for intermediate *Pe* values is shown by a dashed line. The

analysis of forced heat convection in this region can be carried out by some numerical method, and it is the subject of a future investigation.

The most interesting case of the present analysis is that of $Pe \gg 1$, namely, when the effect of the acoustic field is very large. In this case heat conduction is limited to the narrow regions of thickness δ', which are very small, as compared with the distance between the plates, h (see Fig. 2). This thickness is inversely proportional to the square root of Pe. As a result, the enhancement of heat transfer for large Pe is proportional to the square root of Pe, i.e. the average Nusselt number is proportional to the amplitude and the square root of the frequency of the acoustic wave.

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